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REPLY

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McKay (1986) presents results from an analysis of the thermal stability of small comets with dust mantles in the vicinity of the earth's orbit. If the water vapor pressure is contained by the comet's gravitational attraction for the dust mantle he finds that the mantle can be marginally maintained for elliptical orbits with perihelia at 1 A.U. and aphelia ~ several A.U. Because the water vapor pressure is confined only by mantle weight in McKay's treatment, the required mantle thickness is large, ~ tens of meters, for a comet with an interior mass of water of ~ 10^8 gm. Frank et al. (1986a) previously report a lower limit for the tensile strength at fracture of ~ 0.1 dyne/cm^2 for the small comets that is based upon the tidal forces due to the earth's gravitational field. This lower limit on the tensile strength is larger than the central pressure due to gravity by a factor of ~ 200. Thus it is unlikely that the water vapor pressure is balanced by the weight of the mantle but instead by the tensile strength of the comet's mantle and/or its core. We use the present opportunity to discuss the thermal stability of a small comet in the inner solar system in the limit of a thin mantle, $\lesssim$ 1 cm^2, for the conditions of thermal equilibrium at each position along its elliptical orbit around the sun. Such a mantle may be composed of carbon from the dissociation of methane by the solar wind, for example (cf. Frank et al., 1986b).

Consider first the equilibrium temperature and water vapor pressure within a small comet at a heliocentric distance r from the sun. The reflectance of the mantle for the solar spectrum is A and the coefficient of emissivity in the infrared is ε. The temperature T, in degrees K, of the small comet is then given by

$$T = (S(1-A)/4\varepsilon\sigma r^2)^{1/4}$$

$$= 278((1-A)/\varepsilon r^2)^{1/4}$$

where S is the solar constant at 1 A.U., σ is the Stefan-Boltzmann constant, and r is in units of A.U.

Thermal conductivities of the thin mantle and the cometary core of water snow are assumed to be sufficiently large to maintain thermal equilibrium along the comet's orbit. Cooling of the comet by vaporization is insignificant relative to the incident solar energy flux. For a small comet with mass 9 × 10^7 gm and density 0.1 gm/cm^3, the comet's radius a is 600 cm. The solar energy flux on the comet's surface at 1 A.U. is thus 1.5 × 10^{12} ergs/sec (S = 1.35 × 10^6 ergs/cm^2-sec). The loss of heat by vaporization can be evaluated from the range of vaporization rates R, ~ 3 × 10^{10} to 3 × 10^{11} H_2O molecules/cm^2-sec, estimated by Frank et al. (1986b) from solar wind ion measurements. The latent heat of vaporization for water ice at T = 200 K is L = 7.9 × 10^{-13} erg/H_2O molecule (Delsemme and Miller, 1971). Thus the rate of energy loss due to vaporization is in the range of $4\pi a^2 RL \simeq 10^5$ to 10^6 ergs/sec. Comparison with the solar energy flux given above demonstrates that cooling by vaporization is insignificantly low.

The thermal heat capacity of these small comets is also low relative to the solar energy influx and the orbital residence periods near the sun. The specific heat of ice at 200 K is 1.6 × 10^7 ergs/gm-K. The thermal heat capacity of the small comet is (9 × 10^7 gm) × (1.6 × 10^7 ergs/gm-K) = 1.4 × 10^{15} ergs/K. A solar energy influx of 1.5 × 10^{12} ergs/sec, if entirely absorbed by the comet, increases the temperature of the entire cometary mass by 10 K in ~ 10^4 seconds, or several hours. A crude estimate for the thermal lag time for the comet's interior relative to the mantle may be obtained by calculating the energy flow through a 1-meter thick outer shell with a temperature differential ΔT. The thermal conductivity of water snow with density 0.1 gm/cm^3 is ~ 6 × 10^3 ergs/cm-sec-K (Dorsey, 1940). The corresponding energy flow through this spherical shell is ~ $4\pi a^2 \times 6 \times 10^3 \times \Delta T/100$ = 2.6 × 10^8 ΔT ergs/sec. From the above discussion the heat capacitance is ~ 1.4 × 10^{15} ΔT ergs for the entire comet and the time constant is then ~ 10^7 seconds. For water snow with density 1.0 gm/cm^3 this time constant is less by a factor of ~ 10, or ~ 10^6 seconds. Because the dwell times of a small comet near perihelion in the vicinity of 1 A.U. are ~ 10^7 seconds, the validity of the assumption of thermal equilibrium in the comet's interior is dependent upon its density. In order to evaluate the least favorable conditions for thermal stability of the comet we shall assume thermal equilibrium of the mantle with the comet's interior.

The equilibrium vapor pressures and temperatures for a small comet as functions of heliocentric radial distance for various values of $(1-A)/\varepsilon$ are shown in Figure 1. The central pressure due to gravity and the lower limit for tensile strength from considerations of tidal forces are also indicated (Frank et al., 1986a). Tensile strengths at fracture for other possible mechanisms for rupturing the mantle, such as surface charging due to hot plasmas as suggested by Frank et al. (1986a), are more difficult to quantitatively evaluate due to the strong dependence of the electrostatic stress upon the potential and the characteristic curvatures of surface topology (cf. Fechtig et al., 1979). It is noted here that electrostatic disruption of the mantle can be expected to occur at altitudes that are independent of the comet's total mass. Because the dimensions of the water vapor cloud are determined by the speed of the vaporizing molecules

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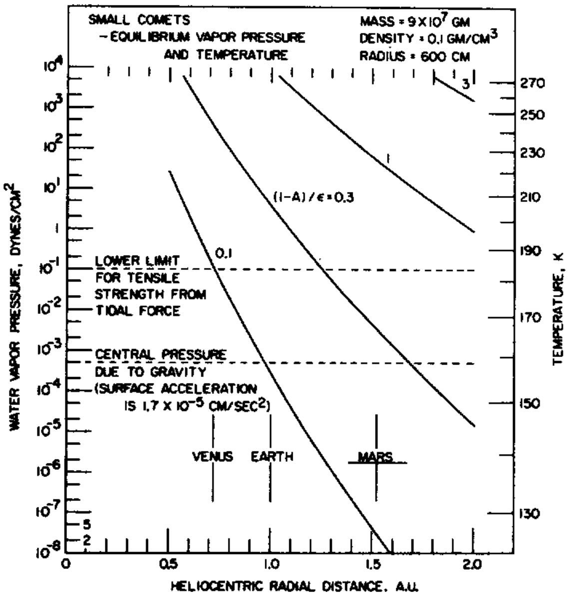

Fig. 1. Equilibrium vapor pressure and temperature for small comets as functions of heliocentric radial distance. The interior of the comet is assumed to be in thermal equilibrium with its thin mantle.

and time of flight into the atmosphere, the dimensions of atmospheric holes will be also generally independent of the total mass. However the minimum mass must be ~ 10^8 gm (Frank et al., 1986a). Thus observations from apogee altitudes with Dynamics Explorer 1 provide the integral fluxes of small comets with masses $\gtrsim 10^8$ gm, but are not useful in determining the mass spectrum of the small comets.

If the mantle is primarily composed of carbon, for example, values for $(1-A)/\varepsilon$ are anticipated to be in the range of ~ 1. The corresponding reflectance A is $\lesssim 0.1$, i.e., the comet is dark. The corresponding vapor pressure is ~ 5×10^3 dynes/cm^2 and the temperature approaches the melting point at 1 A.U (see Figure 1). As a reference for the magnitude of the pressure it is noted that the tensile strength of fresh, powdery water snow at the earth's surface is ~ 5×10^4 dynes/cm^2, or ~ a factor of 10 greater than the vapor pressure at the melting point. Thus for reasonable values of tensile strength and $(1-A)/\varepsilon$ the small comets are thermally stable at the earth's orbit.

As the heliocentric radial distance decreases inside of 1 A.U., the temperature of the small comet reaches and exceeds the melting point of water snow. An examination of Figure 1 shows that for anticipated values of $(1-A)/\varepsilon$, small comet crossing of the Venus orbit is considerably less probable. This conclusion is in agreement with the earlier statements concerning water deposition in the Venus atmosphere by Frank et al. (1986c). However it is possible that a small fraction of the earth-orbit crossing comets reach Venus in a non-equilibrium state due to low thermal conductivity as noted in the above discussion. The heat of fusion of water snow, 3.3 × 10^9 ergs/gm, at the melting point, 273 K, may also prolong the lifetime of the small comets inside the earth's orbit.

The small comet's lifetime against vaporization is large. For the range of vaporization rates 3×10^{10} to 3×10^{11} H_2O molecules/cm^2-sec given by Frank et al. (1986b), and with the assumption that the mantle maintains a constant diameter, these lifetimes are in the range ~ 10^5 to 10^6 years at 1 A.U.

From the above considerations of thermal stability and vaporization in the limiting case of thermal equilibrium of the comet's interior with its thin mantle, we conclude that long-lived small comets in elliptical orbits with perihelia near 1 A.U. are possible.

In summary we reaffirm our previous conclusions (Frank et al., 1986a) that these small comets are distributed in a great disk centered on the sun and lying near the ecliptic plane, and extending from heliocentric radial distances from ~ 1 A.U. to perhaps > 10^4 A.U. These comets follow direct elliptical orbits around the sun. The source of these small comets in the inner solar system is assumed to be due to the gravitational action of the outer planets, passing stars or interstellar clouds on the small comets in more distant orbits.

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